ABSTRACTS

April 26-27, 2014

Steven J. Miller, Williams College

TITLE: *From Sato-Tate distributions to low-lying zeros*

ABSTRACT: Physicists developed Random Matrix Theory (RMT) in the 1950s to explain the energy levels of heavy nuclei. A fortuitous meeting over tea at the Institute in the 1970s revealed that similar answers are found for zeros of L-functions, and since then RMT has been used to model their behavior. The distribution of these zeros is intimately connected to many problems in number theory, from how rapidly the number of primes less than X grows to the class number problem to the bias of primes to be congruent to 3 mod 4 and not 1 mod 4. We report on recent progress on understanding the zeros near the central point, emphasizing the advantages of some new perspectives and models. We end with a discussion of elliptic curves. We'll mix theory and experiment and see some surprisingly results, which lead us to conjecture that a new random matrix ensemble correctly models the small conductor behavior.

Caroline Turnage-Butterbaugh, University of Mississippi

TITLE: *Moments of products of automorphic L-functions*

ABSTRACT: Assuming the Generalized Riemann Hypothesis, we will discuss how to prove upper bounds for moments of arbitrary products of automorphic L-functions and for Dedekind zeta-functions of Galois number fields on the critical line. As an application, we will use these bounds to estimate the variance of the coefficients of these zeta- and L-functions in short intervals.

Frank Thorne, University of South Carolina

TITLE: *Zeroes of L-functions outside the critical strip*

ABSTRACT: Can an L-function have a zero in its region of absolute convergence? If it is an Euler product, then clearly not. But for a wide class of L-functions not given by Euler products, for example the L-function associated to a modular form which is not a Hecke eigenform, we prove that these L-functions can and must have zeroes outside the critical strip.

Our work extends work of Saias and Weingartner, who proved this for the degree 1 case. This is joint work with Andrew Booker.

Dania Zantout, Clemson University

TITLE: *A Geometric Notion of Cusp Forms in Higher Genus*

Abstract: Let Γ be an arithmetic group. Using the theory of Satake compactification of the Siegel variety $\Gamma \setminus hn$, we define a global operator that projects a Siegel modular form to all the rational boundaries of lower degrees. This gives us a generalized notion of cusp forms in higher genus that is in parallel to the notion of elliptic cusp forms.

Nathan McNew, Dartmouth College

TITLE: *Some unconventional results in multiplicative combinatorial number theory*

ABSTRACT: What is the maximal density of an integer sequence that avoids geometric progressions? (It's more than $6/\pi^2$.) What if we look at the largest geometric-progression-free subset of the integers up to N? or modulo N? How many multiplications should we expect to make until a product of random residues modulo N is 0 modulo N? Is it significantly more than the largest prime factor? Which primes are we most likely to encounter as this largest prime factor?

Tom Wright, Wofford College

TITLE: *Variants of Korselt's Criterion*

ABSTRACT: Korselt's Criterion, a necessary and sufficient condition for Carmichael numbers, states that a number n is Carmichael if and only if n is square-free and for each prime p that divides n, $p-1|n-1$. In this talk, we show that under the assumption of a conjecture about the least prime in an arithmetic progression, one can prove that for any $a \in \mathbb{Z}$, there are infinitely n for which $p|n$ implies $p-a|n-a$. This is an improvement of a result of Ekstrom, Pomerance, and Thakur.

Michael Mossinghoff, Davidson College,

TITLE: *A proof from The Magazine*

ABSTRACT: Paul Erdős was well known for labeling a particularly succinct and insightful justification of a theorem its "proof from The Book". We present a problem in number theory, and a solution that may have been drawn from a less ecclesiastical source, which we might dub The Magazine, after the original definition of the word: a storehouse for a large collection of like things. The problem concerns the representation of certain algebraic integers (small negative Pisot and Salem numbers) by very simple polynomials—only 0 and 1 are allowed as coefficients. We give some indication of the storehouse of cases required to tackle the problem, and ask if a Book proof of a more general statement may still lay hidden. This is joint work with Kevin Hare.

James Diffenderfer, Georgia Southern University

TITLE: *Sequences with rational generating functions*

ABSTRACT: Let $\{s_n\}_{n=0}^{\infty}$ be a sequence with the rational generating function $\sum_{n=0}^{\infty} s_n x^n = \frac{f(x)}{g(x)}$, with $deg(f(x)) < deg(g(x))$ and $g(x) \neq 0$. We present a new expression for s_n and examine some applications of this expression. In addition, we present generalized congruences for the sequence s_n.

Jesse Thorner, Emory University

TITLE: *Bounded Gaps Between Primes in Chebotarev Sets*

ABSTRACT: A new and exciting breakthrough due to Maynard establishes that there exist infinitely many pairs of distinct primes p_1, p_2 with $|p_1 - p_2| \leq 600$ as a consequence of the Bombieri-Vinogradov Theorem. We apply his general method to the setting of Chebotarev sets of primes. We study applications of these bounded gaps with an emphasis on ranks of prime quadratic twists of elliptic curves over $\mathbb{Q}$, congruence properties of the Fourier coefficients of normalized Hecke eigenforms, and representations of primes by binary quadratic forms.

Brian Sinclair, University of North Carolina at Greensboro

TITLE: *Enumeration of p-adic Extensions With a Given Ramification Polygon*

Abstract: Let k be a p-adic field. The work of Krasner gives formulae for the number of extensions of k of a given degree and discriminant. Ramification polygons are an extension invariant that helps us to study the wild ramification of an extension. In this talk, we will describe conditions on the ramification polygon of a totally and wildly ramified extension of k of given degree and discriminant. We will use this to provide a constructive specification of Krasner's result to count the number of extensions with a given ramification polygon over k.

Leo Goldmakher, University of Toronto

TITLE: *Bounds on the least quadratic nonresidue*

ABSTRACT: Attaining strong bounds on the least quadratic nonresidue (mod p) is a classical problem, with a history stretching back to Gauss. The approach which has led to the best results uses character sums, objects which are ubiquitous in analytic number theory. I will discuss character sums, their connection to the least nonresidue, and some recent work of myself and J. Bober (University of Bristol) on a promising new approach to the problem.

Michael Filaseta, University of South Carolina

TITLE: 49598666989151226098104244512917

ABSTRACT: A. Cohn showed that if the $d_r d_{r-1} \dots d_0$ is the decimal representation of a prime, then $f(x) = d_r x^r + d_{r-1} x^{r-1} + \cdots + d_0$ is irreducible. Let N be the number in the title. Recently, Sam Gross and the speaker generalized Cohn's result by showing that if $f(x) = \sum_{j=0}^{r} d_j x^j \in \mathbb{Z}[x]$ with $0 \leq d_j \leq N+1$ and $f(10)$ prime, then $f(x)$ is irreducible. Further, we showed, by way of an example, that $N+1$ is best possible. In this talk, we focus on recent further related investigations by Morgan Cole, Scott Dunn and the speaker. In particular, an important property of N will be discussed having connections to factoring integers with the number field sieve.

Rick Farr, University of North Carolina at Greensboro

TITLE: *On Non-integer Stieltjes Constants and Fractional Differentiation*

ABSTRACT: $\gamma_k(a)$ ($k \in \mathbb{N} \cup \{0\}$) are the coefficients of the Laurent series expansion of the Hurwitz zeta function $\zeta(s,a)$ about the point $s = 1$. Kreminski defined these constants for any $k \geq 0$ using the notion of fractional differentiation. We discuss fractional differentiation and give a definition of these constants that is equivalent to Kreminski generalization. From this, we will then prove a conjecture set forth by Kreminski. We will also discuss approximation of these constants using Euler-Maclaurin summation.