100 points. Please write all answers in **ink**. Please use pencil and a straight edge to draw graphs. Allocate your time efficiently.

1. Suppose the messiness of apartments is measured on a scale from 0 to 100, with 0 the cleanest and 100 the messiest. Suppose also that the distribution of apartments by messiness is as shown in the diagram. That is, suppose 10 percent of the apartments lie between 0 and 20, 20 percent between 20 and 40, and so on. Suppose, finally, that all parents tried to teach their children never to let anyone in to see their apartments if they were over 80 on the messiness scale. If such a rule of thumb were widely observed, what would be your best estimate of the messiness index of someone who said, "You can't come in now, my place is a pit"? In a world in which everyone makes use of all available information, would you expect this rule of thumb to be stable? What do you conclude from the fact that people really do sometimes refuse admission on the grounds that their apartments are too messy?

If the general threshold for denying admission is 80, and if those who deny admission are uniformly distributed between 80 and 100, then our best estimate of the messiness index of someone who denies admission will be 90. The threshold will not be stable. Someone whose messiness index is between 80 and 90 has good reason to let people in rather than be assumed to have an index of 90. A threshold of 90 should be unstable for similar reasons, as indeed will any disclosure threshold less than 100. In practice, the fact that some people do refuse to let others see their messy apartments seems to indicate that actually seeing the mess firsthand will be more damaging than having people conclude in the abstract that the apartment is messy.

2. A farmer's hens lay 1000 eggs/day, which he sells for 10 cents each, his sole source of income. His utility function is $U = \sqrt{M}$, where $M$ is his daily income. Each time a farmer carries eggs in from the hen house, there is a 50 percent chance he will fall and break all the eggs. Assuming he assigns no value to his time, is he better off by carrying all the eggs in one trip or by carrying 500 in each of two trips? *(Hint: There are three possibilities when he takes two trips: 1000 broken eggs, 500 broken eggs, and no broken eggs. What is the probability of each of these outcomes?)*

With one trip, $EU_1 = (1/2)\sqrt{100} + (1/2)\sqrt{0} = 5$

With two trips, there are four equally likely outcomes:

<table>
<thead>
<tr>
<th>First Trip</th>
<th>Second Trip</th>
<th>Income</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>0</td>
<td>1000</td>
<td>25%</td>
</tr>
<tr>
<td>500</td>
<td>0</td>
<td>500</td>
<td>25%</td>
</tr>
<tr>
<td>0</td>
<td>1000</td>
<td>0</td>
<td>25%</td>
</tr>
<tr>
<td>0</td>
<td>500</td>
<td>0</td>
<td>25%</td>
</tr>
</tbody>
</table>

* (Hint: There are three possibilities when he takes two trips: 1000 broken eggs, 500 broken eggs, and no broken eggs. What is the probability of each of these outcomes?)

With one trip, $EU_1 = (1/2)\sqrt{100} + (1/2)\sqrt{0} = 5$

With two trips, there are four equally likely outcomes:
EU_2 = (1/4)^2 \sqrt{100} + (1/4)^2 \sqrt{50} + (1/4)^2 \sqrt{50} + (1/4)^2 \sqrt{0} = 2.5 + 1.77 + 1.77 = 6.04. So it is better to take two trips than one, even though the expected number of eggs broken is the same either way. Moral: Don’t put all your eggs in one basket!

3. A firm’s short-run production function is given by

\[ Q = \frac{1}{2}L^2 \quad \text{for } 0 \leq L \leq 2 \quad \text{and} \quad Q = 3L - \frac{1}{4}L^2 \quad \text{for } 2 < L < 7. \]

a. Sketch the production function.

b. Find the maximum attainable production. How much labor is used at that level?

Maximum production (Q=9) occurs at L = 6. [Calculus trained students: To find the maximum, take the first derivative and set it equal to zero and solve for L: dQ/dL = 3 - (1/2)L= 0, which yields L = 6, Q = 9.]

c. Identify the ranges of L utilization over which the marginal product of labor is increasing and decreasing.

For 0 < L < 2, MPL is increasing. For 2 < L < 7, MPL is decreasing.

d. Identify the range over which the marginal product of labor is negative.

\[ MPL < 0 \text{ for } L > 6. \]
4. A firm has access to two production processes with the following marginal cost curves: 
\[ MC_1 = 0.4Q \] and \[ MC_2 = 2 + 0.2Q. \]

a. If it wants to produce 8 units of output, how much should it produce with each process?

The firm minimizes costs when it distributes production across the two processes so that marginal cost is the same in each. If \( Q_1 \) denotes production in the first process and \( Q_2 \) is production in the second process, we have \( Q_1 + Q_2 = 8 \) and \( 0.4Q_1 = 2 + 0.2Q_2 \), which yields \( Q_1 = 6, \) \( Q_2 = 2 \). The common value of marginal cost will be 2.4.

b. If it wants to produce 4 units of output?

Note that for output levels less than 5, it is always cheapest to produce all units with process 1.

![Graph of marginal cost curves MC1 and MC2 with points at (5, 2) and (10, 4)]

Eco 301  
Test 2  
7 November 2008

1. A new motorcycle sells for $9000, while a used motorcycle sells for $1000. If there is no depreciation and risk-neutral consumers know that 20 percent of all new motorcycles are defective, how much do consumers value a nondefective motorcycle?

The expected value of a new motorcycle, \( E_n \), is equal to \( E_n = 9000 = (1-d)X + d \times 1000 \), where \( X \) is the price of a nondefective one and \( d \) is the proportion of defective motorcycles. Therefore, \( 9000 = 0.8X + 0.2(1000) \), which solves for \( X = $11,000 \).

2. A fair coin is flipped twice and the following payoffs are assigned to each of the four possible outcomes:

\( H-H: \) win 20; \( H-T: \) win 9; \( T-H: \) lose 7; \( T-T: \) lose 16.

a. What is the expected value of this gamble?
The expected value of the gamble is given by

\[ EV = \frac{1}{4}(20) + \frac{1}{4}(9) + \frac{1}{4}(-7) + \frac{1}{4}(-16) = +1.5 . \]

b. Suppose your utility function is given by \( U = \sqrt{M} \), where \( M \) is your total wealth. If \( M \) has an initial value of 16, will you accept the gamble in the preceding problem?

\[ EU = \frac{1}{4}\sqrt{36} + \frac{1}{4}\sqrt{25} + \frac{1}{4}\sqrt{9} + \frac{1}{4}\sqrt{0} = \frac{6}{4} + \frac{5}{4} + \frac{3}{4} = 3.5. \]

Your utility without the gamble is \( \sqrt{16} = 4.0 \); so you will not accept the gamble.

3. The following table provides partial information on total product, average product, and marginal product for a production function. Using the relationships between these properties, fill in the missing cells.

<table>
<thead>
<tr>
<th>Labor</th>
<th>Total Product</th>
<th>Average Product</th>
<th>Marginal Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>--</td>
</tr>
<tr>
<td>1</td>
<td>180</td>
<td>180</td>
<td>180</td>
</tr>
<tr>
<td>2</td>
<td>320</td>
<td>160</td>
<td>140</td>
</tr>
<tr>
<td>3</td>
<td>420</td>
<td>140</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>480</td>
<td>120</td>
<td>60</td>
</tr>
</tbody>
</table>

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