Problem solving

Chapter 11
Thought and knowledge

- Problem solving
  - Goal-directed
  - Sequence of operations
  - Cognitive operations
  - Sub-goal decomposition
  - Overcome obstacles along the way

- Decision making
  - Evaluate options
  - Choose between alternatives
How to problem solve

- Well defined vs. ill defined problems
  - Stated goal and clear start/end points?
- Understand the problem
  - Problem space: knowledge at each step
  - Operators: permissible moves
  - Mental representation
- Attention to relevant information
  - Selective attention
- Use problem solving strategy
- Insight
  - Aha! Moment
  - Incubation
Insight

Move 3 circles to get triangle to point down
The ten bowling pins are pointed toward the top of the page. Move any 3 to make the arrangement point down toward the bottom of the page.
Show how to move only 2 pennies in left diagram to yield pattern on the right.
Six pennies

Begin

GOAL

Show how to move only 2 pennies in left diagram to yield pattern on the right.
Insight

- Gestalt psychologists: whole more than collection of parts
  - Insight problems: requires to perceive problem as a whole
  - Wertheimer (1945): requires you to break away from existing associations – see problem in new way
- Kohler (1927): primates
  - Hang food out of reach, but provide objects to use
  - Trial and error; methods varied
- Is insight a gradual process or sudden solution?
Problem solving

- Problem representation
  - Symbols
  - Matrix
  - Tree diagram
  - Graph
Mary is 10 years younger than twice Susan’s age. Five years from now, Mary will be eight years older than Susan’s age at that time.

How old are Mary and Susan?

Representation: ???

Answer: ???
Mary is 10 years younger than twice Susan’s age. Five years from now, Mary will be eight years older than Susan’s age at that time.

How old are Mary and Susan?

Representation: Use symbols = aka algebra!

1. $M = 2S - 10$
2. $M + 5 = S + 5 + 8$
3. $2s - 10 + 5 = s + 5 + 8$
4. $2s - 5 = s + 13$
5. $2s = s + 18$
6. $S = 18$
7. $M = 18 \times 2 = 36 - 10 = 26$
8. $18 + 5 = 23$
9. $26 + 5 = 31$
There are 5 women (Cathy, Debbie, Judy, Linda, and Sonya). Each as a different breed of dog (beagle, retriever, lab, Irish setter, sheepdog), and each has a different occupation (clerk, executive, lawyer, surgeon, teacher). Each also has a different number of kids (0, 1, 2, 3, 4). Given the information below, figure out how many kids the person who owns the sheepdog has.

- The executive owns a retriever.
- The owner of the lab is a surgeon.
- Linda does not own the sheepdog.
- The owner of the retriever has 4 kids.
- Debbie owns the beagle.
- The owner of the sheepdog doesn’t have 3 kids.
- The teacher has no kids.
- Sonya is a lawyer.
- Judy has 1 kid.
- Cathy is a clerk.
- Cathy owns the Irish setter.
Representation: *Matrix*

<table>
<thead>
<tr>
<th></th>
<th>Cathy</th>
<th>Debbie</th>
<th>Judy</th>
<th>Linda</th>
<th>Sonya</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dog</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Occup</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kids</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Matrix: answer

<table>
<thead>
<tr>
<th></th>
<th>Cathy</th>
<th>Debbie</th>
<th>Judy</th>
<th>Linda</th>
<th>Sonya</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dog</td>
<td>Irish setter</td>
<td>Beagle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Occup</td>
<td>Clerk</td>
<td></td>
<td></td>
<td>Lawyer</td>
<td></td>
</tr>
<tr>
<td>Kids</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Matrix: answer

<table>
<thead>
<tr>
<th></th>
<th>Cathy</th>
<th>Debbie</th>
<th>Judy</th>
<th>Linda</th>
<th>Sonya</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dog</td>
<td>Irish setter</td>
<td>Beagle</td>
<td>Lab</td>
<td>Ret.</td>
<td>Sheep-dog</td>
</tr>
<tr>
<td>Occup</td>
<td>Clerk</td>
<td>Teacher</td>
<td>Surgeon</td>
<td>Exec</td>
<td>Lawyer</td>
</tr>
<tr>
<td>Kids</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td><strong>2</strong></td>
</tr>
</tbody>
</table>
Buddhist Monk Problem

Exactly at sunrise, a Buddhist monk set out to climb a tall mountain. The narrow path was not more than a foot or 2 wide, and it wound around the mountain to a beautiful, glittering temple at the peak. The monk climbed the path at varying rates of speed. He stopped many times along the way to rest and to eat. He reached the temple just before sunset. At the temple he fasted and meditated for several days. Then he began his journey back along the same path, starting at sunrise, and walking as before at varying speeds with many stops along the way. However his average speed going down the hill was faster than his average climbing speed.

Prove that there must be a spot along the path that the monk will pass on both trips at exactly the same time of day.
Buddhist Monk: Graph
Representation

- **Restructuring**: Changing the problem’s representation
- Circle problem: If the length of the radius is \( r \), what is the length of line \( x \)?
Problem solving strategies

- **Algorithms**
  - Exhaustive search; step-by-step procedure
  - Always correct

- **Heuristics**
  - Selective search; educated guess
  - “Rule of thumb”

- **Means-end**
  - Goal directed
  - Use “subproblems” and “subgoals”

- **Working backward**
  - Start at goal

- **Analogies**
  - Use similar ideas to get solution
Algorithm or heuristic

Unscramble

S P L O Y O C H Y G

- Algorithm
  - 907,208 combinations
- Heuristic
  - Throw out all YY combinations
  - other heuristics?
Hobbits and Orcs problem

On 1 side of the river are 3 Hobbits and 3 Orcs. They have a boat on their side that can carry 2 creatures at a time across the river. At no point can Orcs out number Hobbits on either side of the river or they will eat the Hobbits. How do you get all 6 creatures across the river?
Hobbits and Orcs: Means-end approach

- The problem can be solved in a minimum of 11 steps
- Means-end analysis: Use sub-goals
- Intermediate steps between initial and goal states
- Trick: have to bring 2 creatures back to original side of river – move away from goal-state.
Tower of Hanoi: *Means end analysis*

- [ ] http://www.mazeworks.com/hanoi/

**Goal:** Move all three rings from the left peg to the right peg:

**Rules:**

1. You can only move one ring at a time
2. You can move only the top ring on a peg
3. You cannot put a larger ring on a smaller ring
How do children (6-7yrs) solve the problem?

Quantitative approach
- Optimal (first) moves, limited errors, initial strategy

Organizational approach
- Constant examination of relationships between moves

Dependent variables:
- First move
- Number of illegal moves
- Number of correct moves to get to goal

Independent variable: successful/unsuccessful child

No relationship between IV and DVs!
- 89% of kids that made illegal move able to solve problem (w/ 91% of illegal moves at start)
Water lilies problem

- Water lilies are growing on Blue Lake. The water lilies grow rapidly, so that the amount of water surface covered by the lilies doubles every 24 hours.
- On the first day of summer, there was just one water lily. On the 90th day of the summer, the lake was entirely covered. On what day was the lake half covered?
Water lilies: *working backwards*

- Use working backwards strategy!
- If doubles every day
  - 90\textsuperscript{th} day 100\%
  - 89\textsuperscript{th} day 50\%
- Answer: 89\textsuperscript{th} day!
Flip it or Dip it

In the game, 3 people flip coins. One person loses and 2 people win each game. The one who loses must double the amount of money that each of the other 2 players has at the time. The 3 players agree to play 3 games. At the end, each has lost 1 game, and each has $8. How much $ did each start out with?
Flip it or Dip it: *Working backward*

- **Answer**
  - * = loser

<table>
<thead>
<tr>
<th>Game</th>
<th>Player1</th>
<th>Player2</th>
<th>Player3</th>
</tr>
</thead>
<tbody>
<tr>
<td>End</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Before game 3</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before game 2</td>
<td></td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>Before game 1</td>
<td></td>
<td></td>
<td>*</td>
</tr>
</tbody>
</table>
Flip it or Dip it: *Working backward*

- Answer
  - * = loser

<table>
<thead>
<tr>
<th>Game</th>
<th>Player1</th>
<th>Player2</th>
<th>Player3</th>
</tr>
</thead>
<tbody>
<tr>
<td>End</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Before game 3</td>
<td>16*</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Before game 2</td>
<td>8</td>
<td>14*</td>
<td>2</td>
</tr>
<tr>
<td>Before game 1</td>
<td>4</td>
<td>7</td>
<td>13*</td>
</tr>
</tbody>
</table>
Radiation problem

Suppose you are a doctor faced with a patient who has a malignant tumor in his stomach. To operate on the patient is impossible, but if the tumor isn’t destroyed the patient will die. A kind of ray, at a sufficiently high intensity, can destroy the tumor. Unfortunately, at this intensity the healthy tissue that the rays pass through on the way to the tumor will also be destroyed. At lower intensities the rays are harmless to healthy tissue, but will not affect the tumor. How can the rays be used to destroy the tumor without injuring the healthy tissue?

Only 10% of participants solve problem
The General story

Consider the following information to assist you with the answer for the above problem. A general who hoped to capture a fortress needed a large number of soldiers but all the roads leading to the fortress were planted with mines. Small groups of soldiers could travel to the fortress safely, since the mines would only be detonated by larger groups. By dividing the army into small groups and sending each group by a different path, the general was able to capture the fortress when the small groups converged from all directions onto the fortress.
Analogy approach

- **Mappings**
  - Attack $\mapsto$ Radiation
  - Fortress $\mapsto$ Tumor
  - Attacking troops $\mapsto$ Rays
  - Small bodies of men $\mapsto$ Weak rays
  - Multiple roads $\mapsto$ Multiple paths
  - Destroy villages $\mapsto$ Damage healthy tissue

- **Holyoak & Thagard (1997)**
  - *Multiconstraint theory*
  - Problem similarity
  - Problem structure (mappings below)
  - Purpose of analogy (goals match)
Analogy approach

- Duncker (1945): radiation problem
  - 10% solve
- Gick and Holyoak (1980): analogy approach
  - Study 1: General hint
    - Radiation problem: 8% solve
    - General story, radiation problem: 75%
  - Study 2:
    - Hint: General story, radiation: 92%
    - No hint: General story, radiation: 20%
  - Study 3: (no hint)
    - Link Fire chief and General story, radiation: 52%
    - Good schema: 91% (only 21% of Ss)
    - Poor schema: 30%
Analogy approach

- What factors help (or hurt) facilitate noticing the relationship and mapping the corresponding parts?
- Hurt: focus on surface features rather than structural features (underlying parts in common)
  - Only lightbulb problem: 10% correct
  - High surface similarity: Radiation problem & solution > lightbulb problem: 81% correct
  - High similarity: fragile glass Lightbulb problem & answer > radiation problem: 69%
  - Low similarity: insufficient intensity lightbulb prob & answer > radiation problem: 33%
- Analogical paradox: in real world problem solving use deeper, more structural features
Experimental methodologies for problem solving

- In-vivo problem solving research
  - Real world problems observed
- Think-aloud protocols
  - Kaplan and Simon’s mutilated checkerboard problem
  - Verbalize thoughts

Can the mutilated board (which used to have 62 squares) be completely covered by placing 31 dominos on the board so that each covers 2 squares?
Influencing factors and bias

- Restructuring
  - Need to represent info in different way
- Mental set (or negative set)
  - Tendency to rely on particular strategy
- Functional fixedness
  - Tendency to use objects in typical ways
- Familiarity
  - Restructure problem so familiar (analogy strategy)
- Creativity
  - Use new information
  - Incubation: set problem aside to think unconsciously
- Expertise
  - Use knowledge to assist problem solving
Nine-dot problem: Mental set

- Connect dots by drawing four continuous straight lines without lifting your pencil from the paper
Nine-dot problem: Solution
The Matchstick Problem

How would you arrange six matches to form four equilateral triangles?
The Matchstick Problem

- Solution to the matchstick problem
- Mental set
- Insight
Water-jug problem

There are 3 jars that each holds a different amount of water. Without any other markers, you need to measure the desired amount. How?

<table>
<thead>
<tr>
<th></th>
<th>Jug A</th>
<th>Jug B</th>
<th>Jug C</th>
<th>Desired</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>21</td>
<td>127</td>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>2.</td>
<td>14</td>
<td>163</td>
<td>25</td>
<td>99</td>
</tr>
<tr>
<td>3.</td>
<td>18</td>
<td>43</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>4.</td>
<td>9</td>
<td>42</td>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>5.</td>
<td>20</td>
<td>59</td>
<td>4</td>
<td>31</td>
</tr>
<tr>
<td>6.</td>
<td>23</td>
<td>49</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>7.</td>
<td>15</td>
<td>39</td>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>8.</td>
<td>28</td>
<td>59</td>
<td>3</td>
<td>25</td>
</tr>
</tbody>
</table>
**Water-jug problem: Mental set**

- **Luchins (1942) Waterjug problem**

<table>
<thead>
<tr>
<th>Jug A</th>
<th>Jug B</th>
<th>Jug C</th>
<th>Desired</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>127</td>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>14</td>
<td>163</td>
<td>25</td>
<td>99</td>
</tr>
<tr>
<td>18</td>
<td>43</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>42</td>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>20</td>
<td>59</td>
<td>4</td>
<td>31</td>
</tr>
<tr>
<td>23</td>
<td>49</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>15</td>
<td>39</td>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>28</td>
<td>59</td>
<td>3</td>
<td>25</td>
</tr>
</tbody>
</table>

80% use prior math

#1-5:
- B - A - 2C
- A - C
- A + C

- A - C
Mair’s 2-string problem

- Goal: tie strings together
- Obstacle: strings too far apart to reach
- Can use other objects in the room (paper, nails, chair, pliers)
- *Functional fixedness*: break out of typical uses for objects
- Only 39% correct in 10min.
Candle problem

- Given: box of tacks, book of matches, candle
- Goal: Attach the candle to the wall and light so it can burn properly
Candle problem

- Functional fixedness: “fixated” on box’s normal function
- Use tacks to attach box to wall
Insight problem
Defeyter & German (2003)

“This is Sam. He’s going on a long journey in his spaceship, and he’s collected together some things for his trip.”

“These are things for writing. This is how Sam writes his name.” (use pencil and pad)

These are things Sam uses for drinking.” (use straw and cup)

“Sam has a pet called Tog. Naughty Tog has run away and got stuck in this tube. Sam can’t leave until Tog is free. Can you show Sam how he can get Tog out? You can use any of Sam’s things, but only 1 at a time.”
Defeyter & German (2003) (exp 1)

- Insight problems and functional fixedness in kids (5-7 years)
- What age does functional fixedness appear?
- Lack of functional fixedness in 5-year-olds
- Consequence of knowing purpose of objects

% selecting target object on first solution attempt

<table>
<thead>
<tr>
<th></th>
<th>5-year-olds</th>
<th>6-year-olds</th>
<th>7-year-olds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function demonstration</td>
<td>60%</td>
<td>40%</td>
<td>40%</td>
</tr>
<tr>
<td>Baseline</td>
<td>80%</td>
<td>80%</td>
<td>95%</td>
</tr>
</tbody>
</table>
Familiarity

- Influence of:
  - Schemas
  - Knowledge
  - How problem is stated - wording
  - Similar to analogy approach

- Problems:
  - Mutilated checkerboard vs. Russian marriage problem
  - Acrobat problem vs reverse acrobat problem
  - Wason selection task
Card problem

- The cards have information about four people sitting at a table. One side of the card is a person’s age and on the other side of the card is what the person is drinking. The rule is that if a person is drinking beer, then the person must be over 21 years old. Select the cards you have to turn over to determine if the rule is violated.

- Average correct: 72%
Card problem #2

- If you borrow my car, you must fill up the gas tank.
- Select the cards you have to turn over to determine if the rule is violated.

Answer:
- Borrowed car (confirm rule)
- Empty gas tank (falsify rule)
Wason four-card problem

- You have 4 cards in front of you. From left to right they are labeled: A, 4, D, 7. If a card has a vowel on one side then it has an even number on the other side. How many cards do you have to turn over to find out whether or not the rule is ever violated?

- Average correct: 15%
- Answer: A: has to be an even # on other side
- Answer: 7 b/c can’t have vowel on other side
Wason four-card problem

- **Abstract task**
  - 53%: know to confirm rule
  - 4%: falsification principle
- **Real-world task**
  - 73%: both correct responses
- **CogLab results**: (# correct rules max = 6)
  - Abstract: global = 1.2
  - Thematic: global = 1.4
- **Conclusion**: Familiarity/knowledge helps you solve the problem
- **Vary familiarity**
  - Reduce % correct if less familiar
Participants shown board then asked to recreate after all pieces were removed.
Expertise

- Expertise allows for:
  - More knowledge
  - Meaningful organization
    - Store more information
  - More time analyzing problems
    - Lesgold (1988): Think-aloud protocol
    - Experts spent more time understanding problem and less time implementing a strategy for solution
  - Automaticity of certain operations
  - No benefit for problems outside of area of expertise
Creativity

- How can we define creativity as a single construct that covers Leonardo da Vinci to Stephen Hawking?
- Production of something original and worthwhile; insightful
- Is creativity a special trait?
  - Or just expertise and commitment?
- Convergent insight: unifying pattern in chaos
- Divergent insight: unique way to think
- Torrance tests of creative thinking
  - Examine diversity, numerosity, and appropriateness of responses
**Activity 1:** Try to improve this stuffed toy rabbit so that it will be more fun to play with. You have 3 minutes.

**Activity 2:** Just suppose that people could transport themselves from place to place with just a wink of the eye or a twitch of the nose. What might be some things that would happen as a result? You have 3 minutes.

**Activity 4:** Add details to diamonds to make pictures out of them. Make the diamond a part of any picture you make. Try to think of pictures no one else will think of. Add details to tell complete stories with your pictures. Give your pictures titles. You have 3 minutes.
Activity 3: Add lines to the incomplete figures to make pictures out of them. Try to tell complete stories with your pictures. Give your pictures titles. You have 3 minutes.
Creativity = intelligence?

- Cattell & Horn’s (1966)
  - Crystalized intelligence
  - Fluid intelligence

- Raven’s progressive matrices
Problem solving hints

- Use different mental representations
- Problem solving strategies
  - Systematic plan
  - Make inferences
  - Create sub-goals
  - Work backwards
  - Search for contradictions
  - Search for relations (analogies)
- Overcome obstacles
  - Functional fixedness
  - Negative or mental set
- Utilize expertise and creativity
  - Increase domain knowledge
  - Increase automaticity to reduce memory load - Practice!
Incubation effects

- **Chain problem: Silveira (1971)**
  - Speak out loud during problem solving
  - Control grp (1 ½ hr): 55%
  - Expmt’l 1 grp (15min, ½ break, 15min): 64%
  - Expmt’l 2 grp (15min 4hr break, 15min): 85%

- **Smith & Blakenship (1989; 1991)**
  - Fixation grp: given incorrect mental set
  - Greater benefit of break

- **Why?**
  - Relax mental set
  - Forget unimportant details
  - Reconstruct information; new perspective
  - Low arousal increases attention span allowing more info to be access simultaneously
Sleep and insight

- Presented with difficult problem
- Groups:
  - Sleep: 8hr
  - Night: 8hr awake
  - Day: 8hr awake