Problem Set 4

1. John is considering renovating the stools at his bar. The production function for new bar stools is given by \( q = 0.1k^{0.2}l^{0.8} \). John would like to provide 10 new bar stools, and he has allocated a budget of $10,000 for the project.

a) John reasons that because capital and labor cost the same amount ($50 per unit), he might as well hire these two inputs in equal amounts. If he proceeds this way, how much of each input will he hire and how much will the renovation cost?

b) Carlos argues that John has forgotten his microeconomics. He asserts that John should choose quantities of inputs so their marginal (not average) productivities are equal. If John opts for this plan instead, how much of each input will he hire and how much will the renovation cost?

3. Suppose that the production function of crayons is conducted at two locations and uses only labor as an input. The production function in location 1 is given by \( q_1 = 10l^{0.5} \) and in location 2 by \( q_2 = 50l^{0.5} \).

a) If a single firm produces crayons in both locations, then it will obviously want to get as large an output as possible given the labor input it uses. How should it allocate labor between the locations in order to do so? Explain precisely the relationship between \( l_1 \) and \( l_2 \).

b) Assuming that the firm operates in the efficient manner described in part (a), how does total output depend on the total amount of labor hired?

5. Do the following production functions have increasing, decreasing, or constant returns to scale? Which ones fail to satisfy the law of diminishing returns?

a) \( q = 4k^2l^2 \)

b) \( q = ak^2 + bl^2 \)

c) \( q = \min(ak, bl) \)

d) \( q = 4k + 2l \)

e) \( q = k^{0.5}l^{0.6} \)

f) \( q = k_1^{0.3}k_2^{0.3}l^{0.3} \)

6. Can the Cobb-Douglas production function be used to portray a production process in which returns to scale are increasing at low input levels and are constant or decreasing at high output levels?
7. Suppose that a firm with the production function \( q = \min(2k, 3l) \) is currently using 6 units of capital and 5 units of labor. What are the marginal products of \( k \) and \( l \) in this case?

8. Graph the TC, ATC, and MC curves for the production function \( q = 3kl \).

10. A firm has a production function with constant returns to scale. Input prices are \( r = 2 \) and \( w = 1 \). The expansion path for this production function at these input prices is a straight line through the origin. When it produces 5 units of output, it uses 2 units of \( k \) and 3 units of \( l \). How much \( k \) and \( l \) will it use when its long-run total cost is 70?

12. A firm has a total cost function given by \( TC = q^3 - 20q^2 + 220q \).
Derive expressions for ATC and MC, and graph these curves.

13. A firm produces output with a Cobb-Douglas production function, \( q = k^\alpha l^\beta \).
Find the expressions for the TC, MC, and ATC.

15. For a firm with the production function \( q = 3 \ln k + 2 \ln l \), find the expression for the optimal ratio of capital to labor.

16. John’s Lawn Mowing Service is a small business that acts as a price taker. The prevailing market price of lawn mowing is $20 per acre. His costs are given by \( TC = 0.1q^2 + 10q \).

a) How many acres should John choose to cut in order to maximize profit?

b) Calculate John’s maximum daily profit.

c) Graph these results and label John’s supply curve.

17. Suppose there are 100 identical firms in a perfectly competitive industry. Each firm has a total cost function of the form \( TC = \frac{1}{300}q^3 + 0.2q^2 + 4q \).

a) Calculate the firm’s supply curve with \( q \) as a function of market price.

b) Calculate the industry supply curve.

c) Suppose market demand is given by \( q = -200p + 8000 \). What will be the equilibrium price-quantity combination?

18. One industry is composed of 100 identical firms, each having total costs given by \( TC = 0.5q^2 - q \).

a) What is the supply curve for each firm? What is the supply curve for the market as a whole?
b) Suppose the demand for the product is given by \( q = 1100 - 50p \). What will be the equilibrium in this market? What will each firm’s total profit be?

21. A firm in a competitive industry has a total cost function of \( TC = 0.2q^2 - 5q \). What is the break-even/shutdown price?