James Bednar

Memo 1: “Not” Confusions

Your memo assignment comes at the end of this handout. First, I want to discuss an important issue concerning logic and everyday English. Before reading on, take a moment to symbolize the following argument:

If you enter the teaching profession, you will not have money for vacations or a nice place or a nice car. If you will not have money for vacations or a nice place or a nice car, then you will not be happy. Therefore, if you enter the teaching profession, you will not be happy. (T, V, P, C, H)

Clear-thinking people can come up with substantially different ways of symbolizing the premises. This is not surprising, since the premises are logically ambiguous and can be reasonably interpreted in two different ways. Consider the first premise. It claims that there is some relationship between your entering the teaching profession and your having money for vacations, a nice place, and a nice car. But what exactly is that relationship? Is the claim that, if you enter the teaching profession, then you will have none of these things? Or is it that, if you enter the teaching profession, then you will not have all of these things? There is a substantial difference between these interpretations. On either interpretation, the argument is valid. The problem, however, is that the interpretations may affect the argument’s soundness. In what follows, I will explicate each interpretation, point out the ways in which the interpretations differ, and diagnose the problem that gives rise to the logical ambiguity exhibited in the premises.

For the sake of simplicity, I will focus only on the first premise; also, to facilitate exposition, I will put the premise in a concrete situation.

Suppose that when Bob graduated from college, a friend said to him: "Bob, if you enter the teaching profession, you will not have money for vacations or a nice place or even a nice car." It is now five years later, Bob is in the teaching profession, and he’s wondering if what his friend said was true or false. She asserted a conditional statement, and a conditional statement is false only in the case that the antecedent is true and the consequent is false. We know the antecedent is true, since Bob has entered the teaching profession. The question, then, is whether the consequent is true or false. To begin, let’s consider some facts about Bob:

1.) Bob does not have money for vacations. ( ~V)
2.) Bob has a nice place. (P)
3.) Bob has a nice car. (C)

Given these facts, on one interpretation, what she said was false, but on a different interpretation, what she said was true. What are these interpretations?

Interpretation I
She may have meant that if Bob enters the teaching profession, then Bob will have none of these things. This interpretation is symbolized by writing:
T → (~V • (~P • ~C)) or equivalently T → ~(V v (~P v ~C))¹ 

Reading back from the symbolization into English, the statement says:

If you enter the teaching profession (T), then 1.) You will not have money for vacations. (~V) and
2.) You will not have money for a nice place. (~P) and
3.) You will not have money for a nice car. (~C)

Now, compare this to the original:

If you enter the teaching profession, then you will not have money for vacations or a nice place or even a nice car.

On this interpretation, the consequent is true just in case Bob has none of these things, and false just in case Bob has at least one of these things. Bob is in the teaching profession, and Bob has at least one of these things; therefore, what she said was false.²

**Interpretation II**

Bob’s friend may have meant that if he enters the teaching profession, then he will not have all of these things. This interpretation is symbolized by writing:

T → ~(V • (P • C)) or equivalently, T → (~V v (~P v ~C))³

Reading back from the symbolization into English, the statement says:

If (T) you enter the teaching profession, then 1.) You will not have money for vacations. (~V) or
2.) You will not have money for a nice place. (~P) or
3.) You will not have money for a nice car. (~C)

On this interpretation, the consequent is true just in case Bob fails to have at least one of these things—money for vacations, a nice place or a nice car—and false just in case Bob has all of these things. Bob is in the teaching profession, but Bob does not have money for vacations;

¹ To see why these two are equivalent, consider DeMorgan’s rules of replacement: ~(p v q) :: ~p • ~q
1. T → ~(V v (~P v ~C))
2. T → (~V • (~P • ~C)) 1 DM (We’ve replaced the first wedge with a dot)
3. T → (~V • (~P • ~C)) 2 DM (Now we’ve replaced the second)

² Given what facts we know about Bob, we can plug in truth-values under the symbolization to show that the statement is false:

<table>
<thead>
<tr>
<th>T</th>
<th>~V</th>
<th>(~V • (~P • ~C))</th>
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<tbody>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
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<tr>
<td>T</td>
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<td>T</td>
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<td>F</td>
</tr>
</tbody>
</table>

3 To see why these two are equivalent, consider DeMorgan’s other rule of replacement: ~(p • q) :: ~p v ~q
1. T → ~(V • (P • C))
2. T → (~V v (~P • ~C)) 1 DM (We’ve replaced the first dot with a wedge)
3. T → (~V v (~P • ~C)) 2 DM (Now we’ve replaced the second)
therefore, what Bob’s friend said was true. If Bob had all three—that is, if on his teacher’s salary he could afford a nice vacation, and a nice place, and a nice car—then what she said would be false. She may, however, only have meant that of these three things there is at least one Bob will have to do without if he enters the teaching profession.

So Which Interpretation is Correct?
Though I doubt that there is a cut-and-dry answer to this question, we may at least hope to make headway by diagnosing the problem. The problem arises from a simple fact about us and about the conventions governing ordinary conversation in English, namely: we tend to take shortcuts. When we talk about a series of items in a conjunctive or disjunctive statement, we often leave out words that would otherwise be repeated several times. For example, someone might say:

I have ham, bacon, and eggs.

In this sentence, the word "and" is left out once, and the phrase "I have" is left out twice. Without cutting corners, the person would have to say:

I have ham and I have bacon and I have eggs.

People don’t normally speak this way. It takes too much time, and why repeat words over and over when they can be left out without sacrificing intelligibility? While this shortcut usually comes off without a hitch, there is a potential for misunderstanding, when multiple occurrences of the word "not" are left out in a conjunctive or disjunctive statement.

The problem is that the scope of "not" becomes unclear: does "not" operate on each conjunct or conjunct individually or does it operate on the disjunction or conjunction as a whole? Going back to what Bob’s friend said, the "not" operates on the disjunction as a whole in interpretation I

\[ \sim (V \lor (P \lor C)) \]

and operates on each disjunct individually in interpretation II

\[ (\sim V \lor (\sim P \lor \sim C)) \]

As we’ve seen, the two are not equivalent. They mean different things, but it’s difficult to tell from what Bob’s friend said whether she meant interpretation I or II. Granted, in this case the logical ambiguity is not that big a deal. But in other cases, it can be very big deal. Consider this little story.

Election 2020
Once upon a time, there was a great race of dwarves, who lived peacefully in a land they called “Adirolf.” Adirolf was a democratic society, with sound laws and benevolent officials. Their greatest elected official was known as the “Commando-in-Chief.” Election year after election year, the dwarves witnessed power change hands from the old to new Commando-in-Chief without a hitch—that is, until the year 2020 a.w. In this momentous year, the election for Commando-in-Chief was extremely close. The dwarves were familiar with close elections; they’d had them before...but none were quite like this one.

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4 Given what facts we know about Bob, we can plug in truth-values under the symbolization to show that the statement is true:

<table>
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<tr>
<th>T</th>
<th>F</th>
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<tbody>
<tr>
<td>T</td>
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<td>T</td>
<td>I</td>
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5 After wheel.
The trouble began shortly after election day when, as a matter of law, the Adiroff election officials called for a statewide manual recount. In most counties, the vote total did not change that much, but in one county—Butterfly County to be exact—they noticed that there were a number of ballots that were rejected by the antiquated ballot-counting machines. The machines rejected these ballots because little chips of paper, known as chad, were not completely punched out of the ballot cards.

Since the rejected ballots could very easily swing the election, the candidates became extremely interested in the fate of these ballots: were they to be thrown into a trashcan or added to a candidate’s vote total? As you might expect, the candidate leading in the election, Rocko, demanded that they be thrown out, while Mojo, the candidate trailing in the election, demanded that they be counted. Rocko, Mojo, and their supporters argued fiercely over the issue. Since both Rocko and Mojo had the utmost respect for the law, they finally turned to the Adiroff State Election Code to settle the dispute. Upon consulting the statutes, they found the following law:

Chad Rejection Law: A ballot is not to be counted if it does not have a chad detached on at least two corners, a hole through which light can pass, or a clear indication of the voter's intent by other means. (For example, rather than punching out the chad, the voter wrote a checkmark over the chad).

Rocko and Mojo agreed that this was the crucial law governing whether or not the disputed ballots were to be counted, for the following facts were true of all the rejected ballots in Butterfly county:

1.) The ballot did not have a chad detached on at least two corners,
2.) Light could be seen through the ballot, and
3.) There was not a clear indication of the voter's intent by other means

Nevertheless, Mojo and Rocko were befuddled by each other's reaction to the newly discovered law.

"Hallelujah!" Mojo rejoiced, "I shall be the next Commando-in-Chief!"

Rocko was stunned. "What have you been smoking, Mojo? The law is clear. I shall be the next Commando-in-Chief!"

As Rocko and Mojo argued back and forth, it became clear that the law, as it stood, was not going to settle this dispute, for the law was logically ambiguous. Rocko and Mojo landed on opposite sides of the ambiguity.

The lawmakers of Adiroff were beside themselves. How could this happen? It was precisely to avoid situations just like this that they had always taken great care to write their laws in the most logically unambiguous way; that’s why they encouraged aspiring lawmakers to study logic. But alas, this law slipped by. It seemed like a perfectly reasonable law when they wrote it, but as the old Adiroff proverb says: "In tranquility, contentment with our laws; in crisis, exposed logical flaws." There was only one thing the dwarves could do, something that had never happened before. That one thing was to settle the election in the Adiroff Supreme Court of Logicians. This was indeed an election like none other.

**Memo Assignment**
You are a clerk for a Supreme Logician. After much study, the Supreme Logicians have all agreed that only the following three principles in the Adiroff Legal System are relevant to settling the dispute between Rocko and Mojo:
Vote Counting Principle:
Ideally, a vote should be counted for a particular candidate if and only if the person who cast the vote wanted it to count for that candidate. (For instance, Dopey’s vote should count for Mojo if and only if Dopey wanted it to count for Mojo.)

Damage Reduction Principle:
If a dispute arises as a result of a logically ambiguous law, Supreme Logicians should prefer the interpretation that avoids the most costly type of error.

Error-Cost Principle:
With respect to the Vote Counting Principle, the omissive error is more costly than the commissive error.

As a clerk for a Supreme Logician, you have been asked to write a memo—no more that two double-spaced pages—arguing that one interpretation of the Chad Rejection Law should be preferred over the other. The Supreme Logician needs you to do three things in the memo.

1.) Articulate as clearly as possible the Mojo’s and Rocko’s interpretations of the Chad Rejection Law. According to standard practice, you take this to involve symbolizing the law according to statement logic. So, symbolize Mojo’s interpretation and symbolize Rocko’s interpretation.

2.) Use the three principles to argue for an interpretation. Do these principles imply Rocko’s interpretation or Mojo’s interpretation? (NOTE: Although there are ways to argue for an interpretation without appealing to the three principles, the Supreme Logician is not interested in them at the moment. Right now, he needs to know what the relevant principles of the Adirolf Legal System imply in this particular case. Your task is to show him.)

3.) So as to avoid this problem in the future, provide a recommendation for rewriting the Chad Rejection Law. It should be equivalent to the legally preferred interpretation (according to your argument), and it should be logically unambiguous.