

On the Dynamics of Generalization

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Shepard (1987) has proposed a universal exponential law of stimulus generalization, yet experimental data are often Gaussian in form. Shepard and others have proposed theories to reconcile the discrepancy, although, as proposed herein, a simple discrete diffusion process may underlie both types of gradient.

Stimulus generalization is the tendency, after experience with a given stimulus, to react in the same way to other, similar stimuli. Shepard (1987) has proposed a universal law for stimulus generalization that deals with relations in a "psychological space," established through multidimensional curve-fitting techniques (MDS). These methods can discover the smallest number of dimensions such that the judged similarity (difference) relations between pairs of stimuli conform to an approximately invariant declining monotonic function of the distance between them. For example, given stimuli *A*, *B*, and *C* with judged difference relations $AB = 1$, $BC = 1$, and $AC = 2$, the psychological space has just a single dimension because the differences among the three stimuli are the same as those among three equidistant points along a line. An invariant similarity function is the reciprocal, given that similarity = $1/\text{difference}$.

Shepard (1987) marshaled both empirical evidence and functional (optimality) arguments in support of an exponential generalization function. However, published comments on his paper noted that the Gaussian function is at least as common an experimental finding as the exponential. Shepard (1988a, 1988b) and Ennis (1988a, 1988b) have both proposed process theories to reconcile the Gaussian and exponential data.

Shepard's suggestion is based on an earlier paper (Shepard, 1958) in which he introduced the concept of *diffusion*: "According to that model, on the removal of an external stimulus, the memory trace of the stimulus spontaneously undergoes a continuing process not only of simple weakening or decay but also of outward diffusion in psychological space" (1988, p. 415). Diffusion, of course, yields the Gaussian response-probability distribution, which will be increasingly broad as the time of testing is delayed after stimulus presentation. In this early model, the exponential function is derived by integration across previous Gaussian traces: "under regularly repeated presenta-

tions of a stimulus, the integrated strength of the accumulated traces left by all preceding presentations approximates an exponential decay function of distance from that stimulus in psychological space" (1988, p. 415).

We now show that a recasting of Shepard's proposal yields an even simpler way to reconcile the exponential (declining, negatively accelerated) and Gaussian gradient forms. It turns out that both are derivable from the process of diffusion alone, with no need for integration or any other long-term process.

The notion of psychological space makes no obvious contact with the possible brain mechanisms that must underlie cognition. Perhaps it is premature to expect such links. Nevertheless, it may be helpful when considering possible generalization processes to take some account of how they might be implemented by neural machinery. The diffusion analysis we present can be done either using the continuous mathematics appropriate for the concept of psychological space or using the discrete mathematics that is more natural for dealing with connectionist networks. We focus on the discrete analysis.

Shepard's theory is designed for an "idealized generalization experiment in which an individual is given a single reinforced trial with a novel stimulus . . . and then is tested with another stimulus from that domain" (1986, p. 60). We deal with the same experimental situation. Suppose that the psychoneural representation of a single stimulus dimension is as a line of units, each with four connections (Figure 1): two bidirectional connections to its two immediate neighbors and an input from a perceptual mechanism that allocates a limited region (e.g., wavelength band) of a single sensory dimension to each unit. When a given region is present in the stimulus, the corresponding unit receives a positive input. Each unit has as its final output an activation strength, x_i , that is its contribution to the measured generalization gradient.

We suppose that the essential property of this very simple, locally connected net is that at each time step, the activation of each unit moves toward the average of its neighbors'. Formally, the change in the strength of the *i*th unit in a series will be given by

$$\delta x_i = \alpha[(x_{i-1} - x_i) + (x_{i+1} - x_i)], \quad 0 < \alpha < .5, \quad (1)$$

where δx_i is the change in x_i from one discrete-time iteration to the next—that is, $x_i(t+1) - x_i(t)$ —and the term in parentheses is the net strength difference between unit *i* and its neighbors, *i* - 1 and *i* + 1. Equation 1 is a discrete-time version of Fick's first

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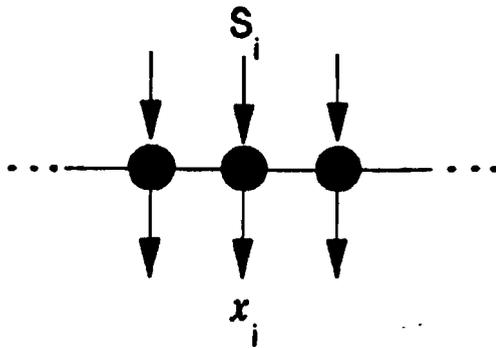


Figure 1. Psychoneural representation of a single stimulus dimension.

diffusion equation (e.g., see Berg, 1983), and α is a diffusion-rate parameter. During any iteration, a unit may also receive stimulus input, so that the net change in activation of a stimulated node will be $\delta x_i + S_i(t)$. Thus, the whole system can be represented as follows:

$$x_i(t+1) = x_i(t)[1 - 2\alpha] + \alpha[x_{i-1}(t) + x_{i+1}(t)] + S_i(t), \quad (2)$$

where $S_i(t)$ is the stimulus input to the i th unit during iteration t .

The behavior of Equation 2 is illustrated in Figure 2, which shows the activation gradient at various points during a total of 30 iterations. A stimulus was present at Unit 9 for the first 5 iterations and not thereafter. The light lines show the growth of a sharply peaked gradient around Unit 9 during the first 5 iterations. The gradient increases in amplitude and moves outward one unit at each iteration. When the number of iterations is moderate (less than 50 or so, depending on the value of α) the generalization gradient is negatively accelerated and closely approximates an exponential function. The gradient collapses to

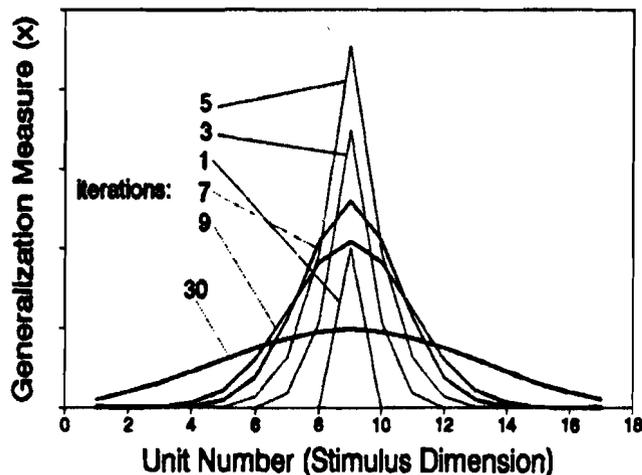


Figure 2. Diffusion in a linear network. (Light lines: Exponential generalization gradient of activation produced by Equation 2 when a stimulus is present [$S_9 = 1$: Iterations 1, 3, and 5]. Heavy lines: Gaussian gradient produced when the stimulus is withdrawn [$S_9 = 0$: Iterations 7, 9, and 30].)

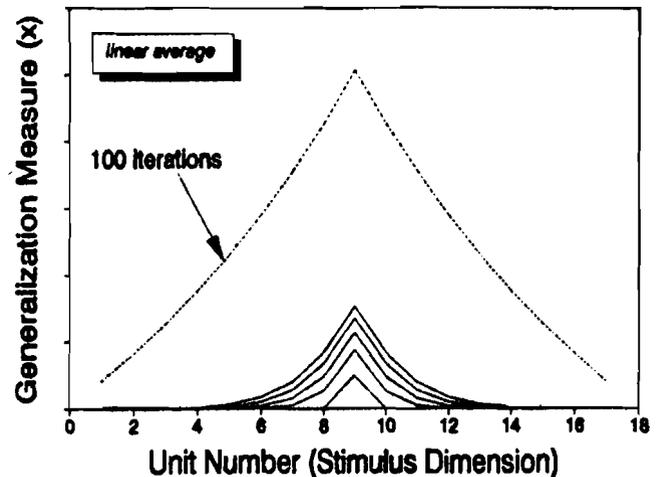


Figure 3. Steady-state form of the linear diffusion gradient (stimulus present). (Units 0 and 18 are assumed clamped at zero activation. Solid lines: Gradient at Iterations 1, 3, 5, 7, and 9. Dotted line: Form of the arithmetic average gradient [Equation 2] after 100 iterations.)

a Gaussian form when the stimulus is withdrawn after Iteration 5, as shown by the gradients at Iterations 7, 9, and 30 (heavy lines).

Thus, a diffusion process alone shows both the negatively accelerated and Gaussian gradient forms, depending on the time delay between target offset and the onset of the test stimulus.

Although the gradient form is approximately exponential when a stimulus is present for a moderate number of iterations, the true steady-state form of the function is not exponential. It approaches a *triangle* under the following two conditions: (a) if the borders (here, the end units) of the net are *absorbing*—that is, their activation value is clamped at 0—or (b) if only a finite region of the activation profile is considered. Case a is illustrated in Figure 3, which shows the effect of prolonged stimulus presentation. The light lines at the bottom show the approximately exponential profile at Iterations 1, 3, 5, 7, and 9; the dotted line at the top shows the profile after 100 iterations. Notice that the gradient approaches a straight-line form rather slowly.

The proof for the steady-state solution is as follows: Let $S_0 = K$, and all others, 0. Then from Equation 2, $\delta x_0 = \alpha(x_1 + x_{-1} - 2x_0) + K$. At equilibrium, $\delta x = 0$. Because the gradient must be symmetrical about Unit 0 we also know that $x_1 = x_{-1}$. Thus, $x_1 - x_0 = -K/2\alpha$, which is also the equilibrium value for $x_2 - x_1$ and, by induction, all other differences. If all first differences are constant, then the function relating x_i to i is a straight line.

The equilibrium form of the gradient predicted by Equation 2 is related to the form of averaging associated with the diffusion process (the term in parentheses in Equation 1). Thus, the term $(x_1 + x_{-1} - 2x_0)$ implies a linear average, because it is 0 when $(x_1 + x_{-1})/2 = x_0$. If we use a geometric instead of an arithmetic average—parenthetical term $(x_{i-1}x_{i+1} - x_i^2)$ —the steady-state gradient resembles the exponential form more closely, as shown in Figure 4. This gradient, like the linear one, also relaxes to an approximately Gaussian form when the stimu-

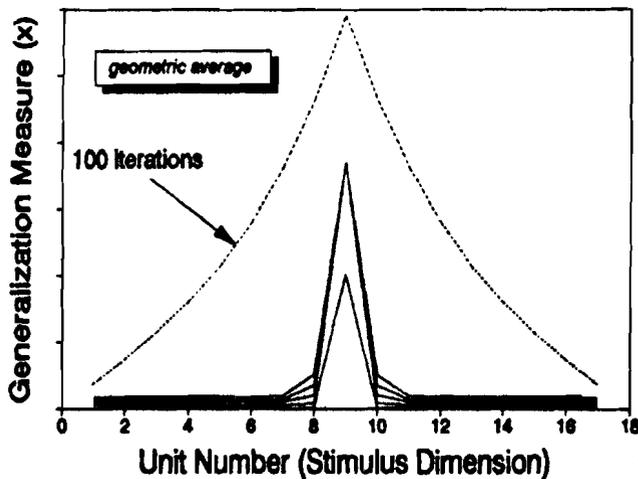


Figure 4. Steady-state form of the geometric diffusion gradient (stimulus present). (Units 0 and 18 are assumed clamped at zero activation. Light lines: Gradient at iterations 1, 3, 5, 7, and 9. Dotted line: Form of the geometric average gradient after 100 iterations. A small resting activation level was assumed in order to avoid multiplying by 0.)

lus input is withdrawn. Thus, the form of the gradient with or without a stimulus present is not very sensitive to the details of the diffusion process.

The most obvious difference between this model and Shepard's (1958) earlier model is that diffusion, in our model, occurs all the time, both when the stimulus is present and when it is not, whereas in Shepard's model diffusion occurs only after the stimulus has ceased. This has implications for the outcome of recognition experiments of the type described by Shepard (1958, p. 247): A subject is shown a certain square (say) for a time T ; after a delay of t , a second square, which may not be the same as the first, is presented. Shepard (1958) reported data showing that the probability of responding "same" to the second square is distributed as a Gaussian function of the size difference between the squares, and the variance increases as a function of t . Our model makes the same prediction, but also makes a similar prediction for the effect of T , the duration of the first stimulus: If interstimulus interval t is short, then recognition accuracy should decrease as T increases. Because the gradient spread increases during (as well as after) stimulus presentation (Figure 2), the larger the value of T , the more subjects should confuse the two stimuli.

The present scheme has both advantages and disadvantages relative to Shepard's (1958) model. The main disadvantage is that it involves short-term memory only, hence cannot explain retrieval effects that depend on stable stimulus representations. Consequently, the experimental prediction just mentioned may fail because any practical test will involve long-term-memory

effects that overlay simple short-term-memory diffusion. The advantages are that the prediction involves just a single well-understood process, relates in a natural way to connectionist theory, and can be implemented by a very simple network. With slight changes, the rate of spread and the rate of decline in peak value of the gradient, which are both determined by the same parameter in simple diffusion, can be dissociated without changing the other useful properties of the process. This scheme can also easily be extended to more than one dimension by postulating nearest neighbor connections in nets of two or three dimensions. It is easy to derive the effect of complex (multiunit) stimuli on the generalization profile.

MDS analysis of experimental data by Shepard and others implies the existence of an invariant form of internal representation in the sense that the shape of the internal gradient does not depend on the physical value of the peak stimulus. The diffusion process has this property in the sense that apart from boundary effects, the shape of the gradient, with respect to the net of units, does not depend on which unit is at the peak. Different forms of equal-generalization contours for multidimensional stimuli can perhaps be accommodated by different diffusion rates in different directions within the net. Although our scheme is inadequate as a complete model for the generalization process (i.e., for the entire process of perceptual recognition and categorization), it may provide useful insights into dynamics and may either form part of, or represent a limiting case of, a more comprehensive theory that is yet to be developed.

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